

**Design of a Solar Chimney to
Generate Electricity
Employing a Convergent
Nozzle**

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Abstract: This paper presents the design of a solar chimney to be used in rural areas of developing countries. The design involves heating air using solar energy and the chimney effect to raise the hot air up the chimneystack. The velocity of the hot air is increased by the use of a convergent nozzle to a velocity suitable to run a wind turbine. The kinetic energy of the hot air is then converted to electricity by a wind turbine.

Keywords: Solar chimney, convergent nozzle.

1. INTRODUCTION.

Providing electricity to remote, rural areas of some developing countries is hampered by the cost of extending the national electricity grid to serve a small population. This problem is perhaps more significant in Botswana than anywhere else. For a country the size of France, with a population of less than 1.5million, the population density in rural areas is too small to justify the cost of extending the grid to provide electricity to the people.

Currently, some remote villages are provided with electricity using diesel generators. The cost of this imported fuel is further increased by transport costs as well as maintenance. Botswana, being a land locked desert country has only solar energy as a viable renewable energy resource. Of the available solar technologies only Photovoltaic (PV) power generation is technically viable. However the high capital cost of imported PV panels and deep cycle, high capacity batteries, as well as replacement cost of batteries, hampers utilization of this technology.

Solar chimney technology holds the promise as solar chimneys can be constructed from locally available materials. Energy storage is achieved by thermal rock energy storage, which is not only cheap, but does not need replacing for the entire life span of the plant.

The concept of solar chimneys tried to date rely on building the chimney very tall to achieve the desired air velocity to run a wind turbine. Such tall chimneys are expensive to construct and will provide a major engineering

challenge, particularly in developing countries. The design of a short solar chimney, which employs the convergent nozzle to increase the air to a suitable speed, is aimed at addressing these problems.

2. OBJECTIVES

In order to complete a design of a solar chimney the following objectives will have to be met:

- Carryout solar radiation calculations incident on a flat plate collector for the worst case scenario (i.e. mid winter) for the whole day, and determine the required collector area, taking into account losses due to reflection at different times of the day.
- Determine the amount of rocks required to store the energy for night use as well as energy storage for overcast days.
- Carryout chimney design calculations for a reasonable height and determine the air velocities resulting from the chimney effect.
- Based on the above, decide on increasing the air velocity by use of a convergent or convergent-divergent nozzle principle as appropriate.
- Carryout a modification of an existing wind turbine to convert the kinetic energy of the air into electrical energy.
- Produce engineering drawings of all the components and the complete assembly.
- Determine test parameters to be measured from the pilot project and source required transducers and data logger.
- Carryout a cost breakdown of all components, and hence determine the cost of electricity.
- Produce a detailed project plan, outlining human resource scheduling and major project milestones.

3. DESIGN CALCULATIONS.

Fig. 1 below shows a pictorial representation of a solar chimney with a pyramid shaped collector.

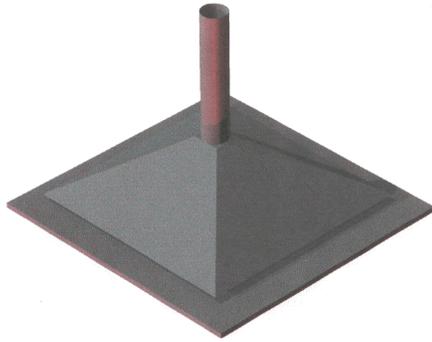


Figure 1. Pictorial Representation of a Solar Chimney.

3.1 Solar Energy Resource.

Solar energy resource was calculated as shown in appendix 1, using the method in [1] through the following steps:

The solar zenith and azimuth angles were calculated at 30 minutes intervals from sunset to sunrise. These were used to determine the angle of incidence on each side of the collector. The beam radiation at each interval was calculated from the angle of incidence, the solar constant and the clearness index for Botswana in mid winter. The clearness index is given in [2] and is based on actual measured data. Beam radiation is shown in fig. 2 below.

From the angle of incidence and the calculated beam radiation, the energy transmitted through the glass cover was determined using Snell's law. Diffuse radiation was determined again using the clearness index. Diffuse radiation is also shown in fig. 2. The net energy transmitted through the glass cover was the sum of the transmitted beam radiation and the diffuse radiation. This is shown in fig. 3.

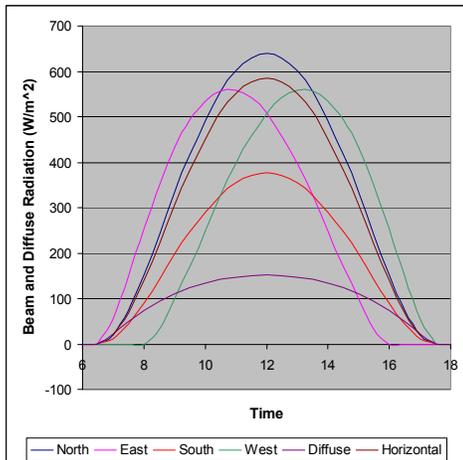


Figure 2. Beam and Diffuse Radiation.

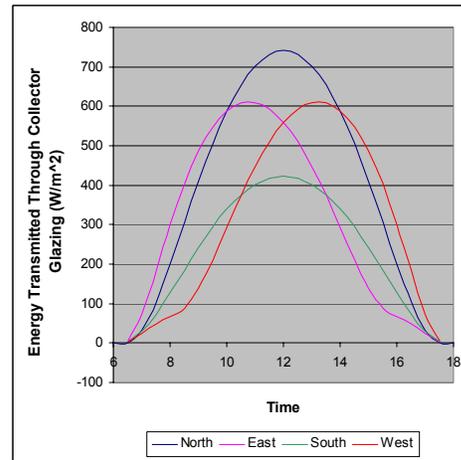


Figure 3. Energy Transmitted Through Collector Glazing.

The daily radiation transmitted through each side (i.e. the area under each curve) is given in table 1 below.

Table 1. Transmitted Radiation.

North Facing Wh/m ²	East Facing Wh/m ²	South Facing Wh/m ²	West Facing Wh/m ²
4541	3650	2751	3650

The area of each side of the pyramid shaped collector is given by:

$$A = \frac{1}{2} * (L * 0.5L / \cos \Delta) \quad (1)$$

The energy collected per m² of base area = $(0.5 * (0.5 * 1) / \cos 30) * (4541 + 3650 + 2751 + 3650) = 4212 \text{ Wh/m}^2$.

3.2 Heat Losses.

The heat losses from the collector were calculated using the method given in [3] through the following steps;

The radiative and convective heat transfer coefficients between the collector and the glass cover were determined to give the resistance between the collector and the cover. The linearised radiative exchange with the sky and the wind related losses were determined to give the cover resistance. The cover resistance and the resistance between the collector and the cover were used to determine the top loss coefficient. From the top loss coefficient, the rate of energy loss per m² of collector base area was determined. Useful energy was calculated as the difference between the energy

transmitted through the collector and the heat losses.

The rate of energy loss is given by:

$$Q_L = U_t A (T_p - T_a) = 6.51 \text{ W/m}^2 \text{ of base area.} \quad (2)$$

$$\text{Energy lost per 24 hour period} = 24 * 6.51 = 156 \text{ Wh/m}^2.$$

$$\text{Useful energy} = 4212 - 156 = 4056 \text{ Wh/m}^2.$$

The calculations for heat losses are shown in appendix 2.

3.3 Chimney Design Calculations.

Consider a chimney 36m high and 4m diameter, chosen due availability of glass reinforced plastic pipes of same diameter in Botswana. These pipes are available in lengths of 12m and can be joined together as they are flanged. Let the ambient temperature (T_a) be 5°C and the hot air temperature (T_h) be 10°C. With no wind turbine inside the chimney the maximum air velocity is:

$$V_{\max} = \sqrt{[2gH(1 - T_a/T_h)]} = 3.53 \text{ m/s.} \quad (3)$$

The proof of this unusual equation is shown in appendix 3.

The mass flow rate is:

$$m = \rho VA = 53.27 \text{ kg/s} \quad (4)$$

Heat flow rate is given by:

$$q = mc_p \Delta T = 267.68 \text{ kW} \quad (5)$$

$$\text{Energy required for 24 hour operation} = 24 * 267.68 = 6424 \text{ kWh.}$$

$$\text{Required collector base area} = 6424 \text{ kWh} / 4056 \text{ Wh} = 1584 \text{ m}^2 \text{ or } 40 \text{ m} * 40 \text{ m.}$$

In order to use the free stream wind speed equation inside the chimney, the diameter of the turbine will be taken as half the diameter of the chimney. The maximum theoretical power output of a wind turbine is given as:

$$P = 2\rho AU^3 a(1-a)^2 \\ = 2 * 1.2 * (0.97^2 * \pi / 4) * 3.53^3 * (1/3) * (1 - (1/3))^2 = 49.24 \text{ W.} \quad (6)$$

The energy over 24 hour period = 49.24 * 24 = 1181.76 Wh. This is a very small output for the given input energy. The reason for such a poor performance is discussed below.

For a chimney with no wind turbine, the total pressure (P_{tot}) is converted into the kinetic energy of the air.

$$\text{K.E.} = P_{\text{tot}} = \frac{1}{2} m V_{\max}^2$$

$$\text{And chimney efficiency } (\eta) = P_{\text{tot}} / q,$$

$$q = mc_p \Delta T$$

$$V_{\max} = \sqrt{(2gH\Delta T / T_h)},$$

$$P_{\text{tot}} = \frac{1}{2} m (2gH\Delta T / T_h),$$

$$\eta = m(gH\Delta T / T_h) / mc_p \Delta T = Hg / c_p T_h$$

g , and c_p are constants, the chimney efficiency is dependant on height and temperature. c_p is in the order of 1005 kJ/kgK and T_h will always be above 273K (i.e. the denominators of the efficiency equation is in the order of 273000 or above). For the efficiency to be increased to a figure close to 100%, the chimney height must be in the order of 273000/9.81 = 27828m. This explains the magic figure of 1000m height currently proposed by solar chimney developers, which will increase the efficiency to about 3.5%. Increasing the chimney height to produce reasonable velocity is too expensive and will provide a major engineering challenge. Fig. 5 below shows the variation of the chimney efficiency with height at constant chimney temperature of 283K. The analysis above shows that although the chimney temperature is above the ambient temperature, it should be kept as low as possible as T_h appears on the denominator of the efficiency equation.

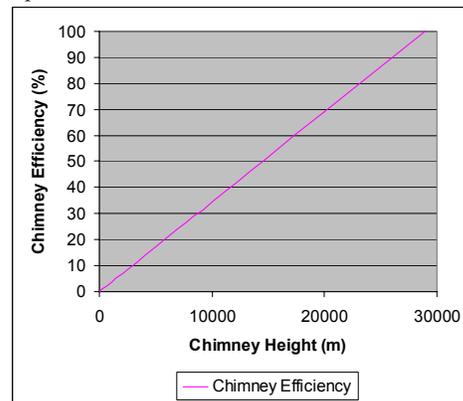


Figure 5. Variation of Chimney Efficiency with Height.

In order to maintain the chimney height at 36m and the require collector area at 1584m², a convergent nozzle will be used to increase air velocity to a suitable 15m/s.

Let the entry conditions to the nozzle be denoted by subscript 1 and the exit conditions be denoted by subscript 2. The entry conditions are known. What is required is the exit (or throat) area, which will give the required velocity of 15m/s. From conservation of mass;

$$A_1\rho_1V_1=A_2\rho_2V_2 \quad (7)$$

Let $\rho_1=\rho_2$, which is reasonable for such a small temperature difference.

$$1.2*3.53*\pi*4^2/4 = 1.2*15*A_2$$

$A_2= 2.96m^2$. Let the diameter of the turbine be half the diameter of the nozzle throat.

The power output $P=2\rho AU^3a(1-a)^2 = 2*1.2*0.97^2*(\pi/4)*15^3*(1/3)(1-(1/3))^2 = 887.8W$.

The energy over 24 hour period = $887.8*24 = 21307.2Wh$, a considerable improvement over the previous 1182Wh.

3.4 Energy Storage.

For continuous operation some of the collected energy has to be stored to be used during the night and cloudy days. Thermal energy can be stored cheaply in rocks, where the stored energy is given by;

$$E=mc_m\Delta T \quad (8)$$

When using the mass specific heat or

$$E= Vc_v\Delta T \quad (9)$$

When using the volumetric specific heat.

The required energy (E) = $6424.32kWh*3.6 = 23.13MJ$.

$c_m=2.8kJ/kgK$ and $c_v=2.9MJ/kgK$

$m=23130/(2.8*5) = 1652kg$

$V = 23130/(2.9*10^3*5) = 1.59m^3$.

The stored energy should allow continuous operation during the night and through periods of overcast weather conditions. For Botswana conditions, the autonomy is given as 7days of overcast weather. The total volume and mass of stones required is therefore;

$m=1652kg*7days = 11564kg$.

$V = 1.59m^3*7days = 11.13 m^3$.

These rocks will be contained in the volume below the collector surface. The volume below the collector surface is given by $V = 1/3$ of base area * height. For a $40m*40m$ base area

pyramid, the height (h) = $(40/2)*\tan30^\circ = 11.55m$

The volume is therefore $40*40*11.55/3=6160m^3$, which is much greater than the required volume for 7 days energy storage. The extra space can be used for seasonal energy storage since the rocks are almost free.

3.5 Turbine Blade Calculations.

Turbine blade calculation were carried out using the method in [9], through the following steps:

The local and tip speed ratios were calculated from the air velocity, the turbine radius and the rotational speed. The blade geometry parameter was calculated from the local speed ratio along the blade span. The blade cord was calculated from the blade geometry parameter. The practical blade cord was determined by drawing a line along the 70% and 90% points of the theoretical blade cord. The inflow angle along the blade span was determined from the local speed ratio. The blade twist angle was calculated from the local inflow angle and the angle of attack along the blade span.

Detailed turbine blade calculations are shown in appendix 4. Table 2 below shows the variation of the blade practical cord and twist angle along the blade span. The theoretical and practical blade cords are represented in fig. 4, while fig. 5 shows the twist angle.

Table 2. Variation of Cord and Twist Angle along Blade Span.

Radius (m)	Twist Angle (β) (degrees)	Blade Cord (m)	Blade Thickness
0.0485	27.13	0.233	0.029
0.097	32.24	0.218	0.026
0.1445	29.62	0.203	0.024
0.194	25.84	0.188	0.023
0.2425	22.27	0.173	0.021
0.291	19.26	0.157	0.019
0.3395	16.79	0.142	0.017
0.388	14.74	0.127	0.015
0.4365	13.05	0.112	0.013
0.485	11.63	0.097	0.012

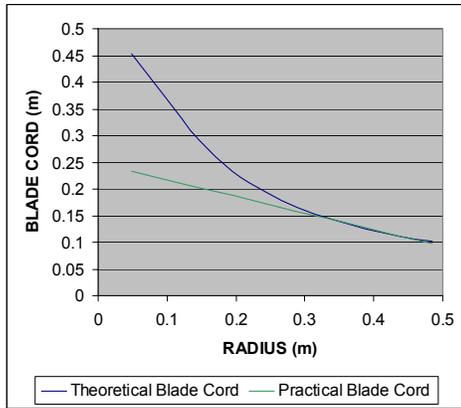


Figure 4. Variation of Blade Cord Along Blade Span.

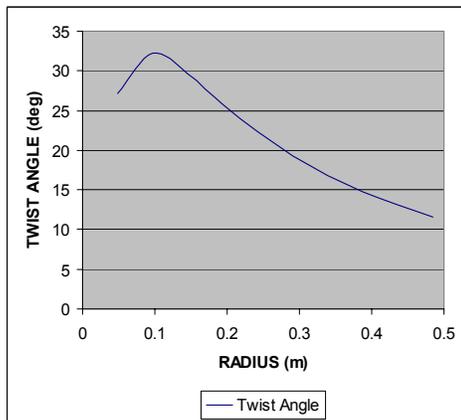


Figure 5. Variation of Twist Angle Along Blade Span.

4. COSTS.

The cost breakdown of a pilot project solar chimney is shown in table 3 below.

Table 3. Cost Breakdown.

ITEM	COST
Glazing (2012m ² of 4mm thick glass inclusive of cut offs)	£12718.35
Angle iron frames	£623.75
Glass reinforced plastic Pipes (3 off)	£750.00
Wiring and accessories	£500.00
Rocks	£300.00
Turbine Blades	£187.50
Generator	£250.00
Nozzle casting (fibre glass)	£150.00
Foundation	£2000.00
Labour	£2500.00
Total	£19979.10

Using the net present value for simplicity, the electrical energy output per day was calculated as 21.3kWhrs per day = 7775kWhrs per annum. Assuming project life span of 20 years, the energy output is 155490kWhrs. The cost of electricity is therefore $\frac{£19979.10}{155490} = £0.128/\text{kWhr}$.

This is compared to the 50kW solar chimney project in Spain where the cost of electricity is given in [5] as £0.62/kWh. In Botswana the average cost of diesel used for generating electricity for remote villages is £0.86/kWhr, and the cost of using centralised PV systems is £0.27/kWhr. Electricity in Botswana is sold at £0.031/kWh.

5. TEST PARAMETERS.

Two VelociCale Air Velocity meters (model 8345) will be used to measure the air velocity just before the wind turbine and some distance upstream. These velocity meters are hot wire anemometers capable of measuring velocities in the range 0 to 30m/s and temperature in the range of -17.8 to 93.3°C. The duct size ranges are 1cm to 1m and 1m to 2.55m.

The electrical output will be measured using a power meter currently available at Botswana Technology Centre.

Two temperature K-type probes will be used to measure the rock temperature one on the surface and the other imbedded in one of the rocks. Another temperature probe will be used to monitor the ambient temperature.

A Delta-T Devices DL2e data logger will be used to sample and store all the data. Data sampling will be at 5 minutes intervals and averaged at 30 minutes intervals. The data logger is also available.

6. PROJECT PLAN AND DRAWINGS.

The project plan for this prototype is shown in appendix 5. The plan shows that the project will take a little more than 1 month to complete. The task scheduling is such that one task is finished in order to start a new task, with no delays between the tasks, except for a 2 days delay after completing the foundation. This delay is to allow the concrete to cure before installing the chimney. Tasks without interdependency are allowed to run concurrently.

The drawings are shown in appendix 6.

7. DISCUSSION.

The design of this pilot project was cautiously limited to the wind turbine rated speed of 15m/s. Fig. 6 below shows the variation of the cost of electricity with air velocity when the collector area and chimney height were maintained at 40m*40m and 32m respectively, and the nozzle throat varied to increase the velocity. If the air velocity can be increased to about 25m/s, the cost of electricity will be reduced considerably. Considering that the turbine will be installed inside a chimney, the risk of injury to people or damage to property in case of blade failure is eliminated. This should allow operation of the turbine at a higher air velocity than 15m/s.

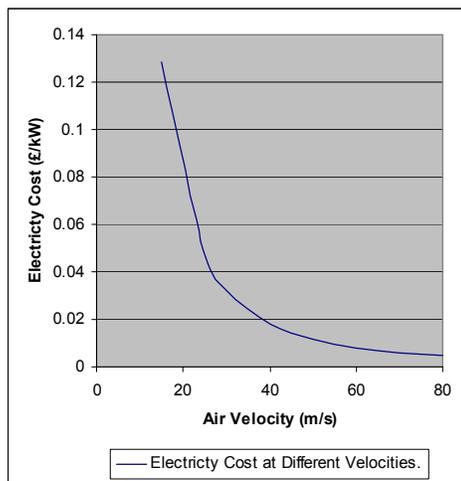


Figure 6. Electricity Cost Vs Air Velocity.

For the design of this prototype the theoretical output of the chimney was limited to about 890W in order to minimise the research and development cost. In real life situations, the solar chimney will have to be made larger than this to be able to provide electricity to remote villages. It is anticipated the multiple plants in the region of 5kW per chimney will be required, depending on the size of the village.

Small solar chimneys will find use in other applications such as remote telecommunications repeater stations, remote industrial setups and water pumping.

8. CONCLUSIONS

The solar radiation calculations show that the useful energy for a mid winter day in Botswana is 4056Wh/day per square meter of

base area for a pyramid shaped collector with slope angle of 30°.

For a solar chimney 36m high and 4m diameter, the air velocity was found to be 3.53m/s and the maximum theoretical power output of 49.24W. Using a convergent nozzle, the velocity was increased to 15m/s and the power increased to 887.8W.

Turbine blade calculations produced a design of a NACA4412 3-blade rotor with a diameter of 0.97m.

The volume of rocks required to store the energy was found to be 11.31m³, which is far less than the available volume of 6160m³ below the pyramid collector. The extra available volume can be used for seasonal energy storage since the rocks are almost free.

Using present value, the cost of electricity for this prototype was calculated as £0.13/kWh, which is better than currently available technologies for Botswana situation. The cost is expected to drop for larger plants.

The project plan shows that the prototype will take a little more than a month to complete, inclusive of the project report.

9. REFERENCES.

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APPENDIX 1. SOLAR ENERGY RESOURCE CALCULATIONS

Solar Energy Resource.

The relevant angles to be used on solar energy resource calculations are shown in figure 1 below.

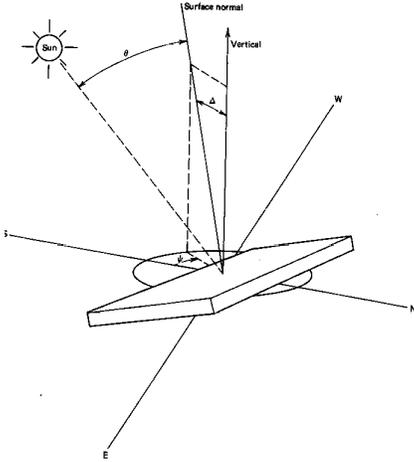


Figure 1. Sun angles on collector surface.

The collector for the solar chimney will be in the shape of a pyramid, with the chimney sticking out from the middle of the pyramid. The choice of a pyramid structure is to allow dirt to slide off the collector cover (as compared to a horizontal collector) as well as to allow the air to enter the chimney from different sides (as compared to a collector sloping to one side only).

For Gaborone, the longitude is 25.55° east and latitude (L) = 24.55° south. Colatitude (L') = $90^\circ - 24.55^\circ = 65.55^\circ$.

$$\delta = 23.45 \sin[360/365 * (\text{DoY} + 284)] \quad (1.1)$$

$\delta = 23.45 \sin[360/365 * (172 + 284)] = 23.45^\circ$, where δ is the solar declination and DoY is the day of the year = 172 for mid winter in Botswana.

$$\cos \omega_s = -\tan \delta \tan L \quad (1.2)$$

$$\cos \omega_s = -\tan 23.45 \tan -24.55 = 0.1981.$$

$\omega_s = \pm 78.57^\circ$, where ω_s is the hour angle at sunset and sunrise and L is given as a negative number for the Southern Hemisphere.

$$T_{\text{sunrise}} = ((-\omega_s * 60/15) + 720)/60 \quad (1.3)$$

$$T_{\text{sunrise}} = ((-78.57 * 60/15) + 720)/60 = 5.07$$

$$\text{And } T_{\text{sunset}} = ((\omega_s * 60/15) + 720)/60 \quad (1.4)$$

$T_{\text{sunset}} = ((78.57 * 60/15) + 720)/60 = 17.24$, where the time is given as a fraction of an hour instead of hours and minutes.

The hour angle ω is given by;

$$\omega = \pm (360^\circ * t) / 24 \text{hrs.} \quad (1.5)$$

Where t is the number of hours before (negative) or after (positive) solar noon.

At solar noon, the hour angle $\omega = \pm (360^\circ * 0) / 24 \text{hrs} = 0^\circ$.

The codeclination D' is given by;

$$D' = \cos^{-1}(\sin 23.5 (\sin 360 * n / 365.25)) \quad (1.6)$$

Where n is the number of days after the vernal equinox = 86 for mid winter in Botswana.

$$D' = \cos^{-1}(\sin 23.5 (\sin 360 * 86 / 365.25)) = 89.66$$

$$\mu_0 = \cos Z = \cos D' \cos L' + \sin D' \sin L' \cos \omega \quad (1.7)$$

Where Z is the solar zenith angle.

$$\mu_0 = \cos 89.66 * \cos 65.55 + \sin 89.66 \sin 65.55 \cos 0 = 0.9128.$$

$$Z = \cos^{-1} 0.9128 = 24.11^\circ.$$

$$\tan A = \sin D' \sin \omega / (\sin D' \cos L' \cos \omega - \cos D' \sin L') \quad (1.8)$$

Where A is the solar azimuth angle.

$$\tan A = \sin 89.66 \sin 0 / (\sin 89.66 \cos 65.55 \cos 0 - \cos 89.66 \sin 65.55) = 0^\circ.$$

$$A = \tan^{-1} 0 = 0^\circ.$$

$$\mu = \cos \theta = \cos Z \cos \Delta + \sin Z \sin \Delta \cos (A - \psi) \quad (1.9)$$

Where Δ is the collector slope angle = 30° , θ is the obliquity angle and ψ is the azimuthal collector orientation = 0° for north facing collector at the Southern Hemisphere.

$$\mu = \cos 24.11 \cos 30 + \sin 24.11 \sin 30 \cos 0 = 0.9947$$

$$\theta = \cos^{-1} 0.9947 = 5.90^\circ$$

$$F_{\text{beam}} = S \mu e^{-\tau/\mu_0} \quad (1.10)$$

Where S is the solar constant = 1352 W/m^2 and τ is the clearness index = 0.68 for mid-winter Botswana conditions.

$$F_{\text{beam}} = 1352 * 0.9947 * e^{-0.68/0.9128} = 638.46 \text{ W/m}^2$$

Diffuse Radiation.

$$\gamma^\pm = 0.5(C-A) \pm 0.5[(C+A)-4BD]^{1/2} \quad (1.11)$$

where $A = (2 - \omega_0)/2\mu_0$, $B = \omega_0$, $C = (2 - \omega_0)$ and $D = \omega_0/2\mu_0$. $\omega_0 = 0.5$ = the single scattering albedo.

$$A = (2 - 0.5)/(2 * 0.9128) = 0.8217, \quad B = 0.9128, \quad C = (2 - 0.5) = 1.5 \quad \text{and} \quad D = 0.5/(2 * 0.9128) = 0.2739$$

$$\gamma^+ = 0.5(1.5 - 0.8217) \pm [(1.5 + 0.8217)4 * 0.9128 * 0.2739]^{1/2}$$

$$\gamma^+ = 1.4394 \quad \text{and} \quad \gamma^- = -0.7611$$

$$G = -[(\gamma^+ + A - BR)/(\gamma^+ + A - BR)] \exp((\gamma^- - \gamma^+) \tau)$$

$$(1.12)$$

Let the reflectivity $R = 0.5$.

$$G = -[(-0.7611 + 0.8217 - (0.5 * 0.2))/(1.4394 + 0.8217 - (0.5 * 0.2))] e^{(-0.7611 - 1.4394)0.68} = 0.000717$$

The downward flux $F_{\text{diffuse}}^{\downarrow}$ is given by;

$$F_{\text{diffuse}}^{\downarrow} = \mu_0 S [(1/(G/(1+G)) \exp(\gamma^+ \tau) + (1/(1+G)) \exp(\gamma^- \tau) - \exp(-\tau/\mu_0)]$$

$$(1.13)$$

$$F_{\text{diffuse}}^{\downarrow} = 0.9128 * 1352 [(0.000717/(1+0.000717)) * e^{(1.4398 * 0.68)} + (1/(1+0.000717)) e^{(-0.7611 * 0.68)} - e^{(-0.68/0.9128)}] = 151.86 \text{ W/m}^2$$

The upward flux $F_{\text{diffuse}}^{\uparrow}$ is given by;

$$F_{\text{diffuse}}^{\uparrow} = R \mu_0 S [(1/(G/(1+G)) \exp(\gamma^+ \tau) + (1/(1+G)) \exp(\gamma^- \tau)]$$

$$(1.14)$$

$$F_{\text{diffuse}}^{\uparrow} = 0.5 * 0.9128 * 1352 [(0.000717/(1+0.000717)) * e^{(1.4398 * 0.68)} + (1/(1+0.000717)) e^{(-0.7611 * 0.68)}] = 147.49 \text{ W/m}^2$$

For an inclined surface, the diffuse flux is dependent only on the slope angle and not the orientation relative to the azimuth.

$$F_{\text{diffuse}} = [(1 + \cos \Delta)/2] F_{\text{diffuse}}^{\downarrow} + [(1 - \cos \Delta)/2] F_{\text{diffuse}}^{\uparrow} \quad (1.15)$$

$$F_{\text{diffuse}} = [(1 + \cos 30)/2] * 151.86 + [(1 - \cos 30)/2] * 147.49 = 148.46 \text{ W/m}^2$$

Repeating the calculation for beam and diffuse radiation at different times between sunrise and sunset, and for surfaces orientated to the east, south and west gives the results shown in figure 1 below.

Global Radiation.

$$H_{\text{global}} = H_{\text{diffuse}} + H_{\text{beam}}$$

$$H_{\text{global}} = 148.46 + 638.46 = 787.2 \text{ W/m}^2$$

Repeating these calculations for other times from sunrise to sunset, and for the collector sides facing east, south and west produced the results shown in figure 2 below.

Energy Lost Due to Reflection on Cover Material.

$$\theta_1 = \theta = 5.90^\circ$$

$$\theta_2 = \sin^{-1}(\sin \theta_1 (n_1/n_2)) \quad (1.16)$$

$$\theta_2 = \sin^{-1}(\sin 5.90 * (1.0003/1.5)) = 3.93^\circ$$

The reflectance (ρ) is given by;

$$\rho = \frac{1}{2} [(\sin^2(\theta_2 - \theta_1)/\sin^2(\theta_2 + \theta_1)) + (\tan^2(\theta_2 - \theta_1)/\tan^2(\theta_2 + \theta_1))] \quad (1.17)$$

$$\rho = \frac{1}{2} [(\sin^2(3.93 - 5.9)/\sin^2(3.93 + 5.9)) + (\tan^2(3.93 - 5.9)/\tan^2(3.93 + 5.9))] = 0.0399$$

The net transmission τ_r (assuming there is no absorption by the glazing material) is given by;

$$\tau_r = (1 - \rho)/(1 + \rho) \quad (1.18)$$

$$\tau_r = (1 - 0.0399)/(1 + 0.0399) = 0.9232$$

$$\text{The radiation transmitted through the glazing } F_{\text{tglass}} = F_{\text{beam}} * \tau_r + F_{\text{diff}} \quad (1.19)$$

$F_{\text{glass}} = 638.45 * 0.9232 + 148.74 = 738.15 \text{ W/m}$ for the north facing side at solar noon. Repeating these calculations for the east, west and south facing sides throughout the day produced the results shown in figure 3 below.

APPENDIX 2. HEAT LOSSES CALCULATIONS.

Heat Losses.

Let the collector plate temperature be 50°C and the ambient temperature be 5°C.

The linearised radiation coefficient h_r is given by;

$$h_r = \sigma(T_p + T_c)(T_p^2 + T_c^2)/(1/\epsilon_p + 1/\epsilon_g - 1) \quad (2.1)$$

$$h_r = 5.67 \cdot 10^{-8} \cdot (323 + 298) \cdot (323^2 + 298^2)/(1/0.97 + 1/0.07 - 1) = 0.475.$$

Where σ is the Stephan-Boltzmann constant, T_p and T_c are the plate and cover temperatures respectively and ϵ_p and ϵ_g are the plate and cover emmittances respectively. The cover temperature is chosen as 25°C (i.e. between the ambient and the plate temperatures).

The convective heat transfer coefficient h_c is given by;

$$h_c = [0.06 - 0.017(s/90)]kL(g\Delta T/Tv^2)^{1/3} \quad (2.2)$$

$$h_c = [0.06 - 0.017 \cdot (30/90)] \cdot (0.026 \cdot 0.01) \cdot (9.81 \cdot 45 / (323 \cdot (1.5 \cdot 10^{-5})^2))^{1/3} = 0.0295.$$

Where k is the thermal conductivity of air, L the distance between the plate and the cover, v the kinematic viscosity and T the air temperature.

$$R_2 = 1/(h_r + h_c) \quad (2.3)$$

$$R_2 = 1/(0.0295 + 0.475) = 1.98.$$

Wind loss coefficient is given by

$$h_w = 5.7 + 3.8V \quad (2.4)$$

$$h_w = 5.7 + (3.8 \cdot 3) = 64.98,$$

Where V is the average wind speed = 3m/s at 2m height for Botswana conditions.

$$T_{sky} = 0.0552T_{air}^{3/2} \quad (2.5)$$

$$T_{sky} = 0.0552 \cdot 278^{3/2} = 255.86K.$$

The radiative heat transfer coefficient referenced to the air temperature.

$$h_r = \sigma\epsilon_g(T_c + T_{sky})(T_c^2 + T_{sky}^2)[(T_c - T_{sky})/(T_c - T_a)] \quad (2.6)$$

$$h_r = 5.67 \cdot 10^{-8} \cdot 0.97 \cdot 298 + 255.86 \cdot (298^2 + 255.86^2) \cdot [(298 - 255.86)/(298 - 278)] = 9.9.$$

$$R_3 = 1/(h_r + h_w) \quad (2.7)$$

$$R_3 = 1/(9.9 + 64.98) = 0.0134.$$

The overall loss coefficient

$$U_t = 1/(R_2 + R_3) \quad (2.8)$$

$$U_t = 1/(1.98 + 0.0134) = 0.5011.$$

The rate of energy loss

$$Q_L = U_t A (T_p - T_a) \quad (2.9)$$

$$Q_L = 0.5011 \cdot 4 \cdot 0.5 \cdot (0.5/\cos 30) \cdot (323 - 278) = 6.51 \text{ W/m}^2 \text{ of base area.}$$

$$\text{Energy lost per 24 hour period} = 24 \cdot 6.51 = 156.24 \text{ Wh/m}^2.$$

$$\text{Useful energy} = 4212.10 - 156.24 = 4055.86 \text{ Wh/m}^2.$$

APPENDIX 3. PROOF OF CHIMNEY EQUATION.

The proof of air flow inside a chimney is covered in [6] and is given as follows:

From Torricelli equation $V=(2gh)^{1/2}$. (3.1)

If the specific gravity of the general mass of cooler air surrounding the chimney is called unity, and that of hot air s_h , the net buoyant head responsible for velocity is

$h=(1-s_h)m$, (3.2)

where m is the chimney height.

The value for s_h due to temperature can be expressed in terms of the ratio of absolute temperatures, $s_h = T_c/T_h$, (3.3)

where subscripts c and h refer to the cooler and hotter air masses. The formula

$V=(2gh)^{1/2}$ then becomes $V=(2gm(1-T_c/T_h))^{1/2}$ (3.4)

APPENDIX 4. TURBINE BLADE CALCULATIONS.

Selecting a 6 pole 3-phase synchronous speed generator, the rotor speed at 50hz is 1500 rpm. The tip speed ratio for a 0.97-meter diameter rotor is given by;

$$\lambda = \Omega R / U_{\infty} \quad (4.1)$$

$$\lambda = 1500 * 0.485 * \pi / 60 * 15 = 5.08$$

For a NACA0012 blade design, which is chosen for this project because it allows ease of manufacture due to its flat high pressure side, selecting the lift coefficient (C_l) of 0.7 and angle of attack (α) of 3° for optimum operation at constant speed. For optimum operation the flow induction factor (a) is $1/3$.

$$\sigma_t \lambda C_l = (8/9) / \sqrt{[(1-1/3)^2 + \lambda^2 \mu^2 (1 + (2/9)(\lambda^2 \mu^2))^2]} \quad (4.2)$$

$$\text{At } 0.1 * R, \mu = r/R = 0.0485/0.485 = 0.1.$$

$$\lambda^2 \mu^2 = 0.1^2 * 5.08^2 = 0.258.$$

$$\sigma_t \lambda C_l = (8/9) / \sqrt{[(1-1/3)^2 + 0.258 (1 + (2/9)(0.258))^2]} = 0.768 = \lambda C_l N c / 2\pi r$$

For a rotor with number of blades (N) = 3

$$c/R = 0.768 * 2 * \pi / 4.19 * 0.7 * 3 = 0.453.$$

$$\tan \phi = (1-1/3) / [\lambda \mu (1 + (2/(3\lambda \mu^2)))] \quad (4.3)$$

$$\tan \phi = (1-1/3) / [0.1 * 5.08 (1 + (2/(3 * 0.1 * 5.08^2)))] = 0.580.$$

$$\phi = \tan^{-1} 0.580 = 30.13^\circ.$$

$$\beta = \phi - \alpha \quad (4.4)$$

$$\beta = 30.13 - 3 = 27.13^\circ.$$

Where β is the blade twist angle, ϕ is the local inflow angle and α is the angle of attack.

The blade thickness for a NACA4412 blade is given as 12% of the cord.

Repeating these calculations for other positions along the blade from root to tip yields the results shown in table 1 below.

Table 1. Variation of Cord, thickness and Twist Angle along Blade Span.

Radius (m)	Twist Angle (β) (degrees)	Blade Cord (m)	Blade Thickness
0.0485	27.13	0.233	0.029
0.097	32.24	0.218	0.026
0.1445	29.62	0.203	0.024
0.194	25.84	0.188	0.023
0.2425	22.27	0.173	0.021
0.291	19.26	0.157	0.019
0.3395	16.79	0.142	0.017
0.388	14.74	0.127	0.015
0.4365	13.05	0.112	0.013
0.485	11.63	0.097	0.012

APPENDIX 5. PROJECT PLAN.

ID	Task Name	Duration	Start	Finish	Gantt Chart						
1	SOLAR CHIMNEY PROJECT	26 days	Mon 4/1/02	Mon 5/6/02	<p>The Gantt chart displays the project schedule with the following task dependencies and resource assignments:</p> <ul style="list-style-type: none"> Task 1 (Start): Mon 4/1/02 to Mon 4/1/02. Resource: construction company. Task 2 (Construct chimney founda): Mon 4/1/02 to Tue 4/2/02. Resource: construction company. Task 3 (Install chimney): Thu 4/4/02 to Thu 4/4/02. Resource: construction company. Task 4 (Fabricate frames): Mon 4/1/02 to Wed 4/3/02. Resource: technician, 2 labourers. Task 5 (Install frames): Thu 4/4/02 to Fri 4/5/02. Resource: technician, 2 labourers. Task 6 (Fabricate turbine blades): Mon 4/8/02 to Tue 4/9/02. Resource: engineer, technician. Task 7 (Fabricate nozzle): Wed 4/10/02 to Thu 4/11/02. Resource: engineer, technician. Task 8 (Assemble turbine/nozzle): Fri 4/12/02 to Fri 4/12/02. Resource: engineer, technician. Task 9 (Install turbine/nozzle asse): Fri 4/12/02 to Fri 4/12/02. Resource: engineer, technician, 2 labourers. Task 10 (Place rocks under frames): Mon 4/8/02 to Tue 4/9/02. Resource: technician, 2 labourers. Task 11 (Cut glass): Wed 4/10/02 to Thu 4/11/02. Resource: technician, 2 labourers. Task 12 (Install glass cover): Fri 4/12/02 to Mon 4/15/02. Resource: technician, 2 labourers. Task 13 (Install transducer and logg): Tue 4/16/02 to Tue 4/16/02. Resource: engineer, technician. Task 14 (Tests): Wed 4/17/02 to Thu 4/25/02. Resource: engineer, technician. Task 15 (Produce Report): Fri 4/26/02 to Mon 5/6/02. Resource: engineer, technician. Task 16 (Finish): Mon 5/6/02 to Mon 5/6/02. Resource: engine. 						
2	Start	0 days	Mon 4/1/02	Mon 4/1/02							
3	Construct chimney founda	2 days	Mon 4/1/02	Tue 4/2/02							
4	Install chimney	0.5 days	Thu 4/4/02	Thu 4/4/02							
5	Fabricate frames	3 days	Mon 4/1/02	Wed 4/3/02							
6	Install frames	2 days	Thu 4/4/02	Fri 4/5/02							
7	Fabricate turbine blades	2 days	Mon 4/8/02	Tue 4/9/02							
8	Fabricate nozzle	2 days	Wed 4/10/02	Thu 4/11/02							
9	Assemble turbine/nozzle	0.5 days	Fri 4/12/02	Fri 4/12/02							
10	Install turbine/nozzle asse	0.5 days	Fri 4/12/02	Fri 4/12/02							
11	Place rocks under frames	2 days	Mon 4/8/02	Tue 4/9/02							
12	Cut glass	2 days	Wed 4/10/02	Thu 4/11/02							
13	Install glass cover	2 days	Fri 4/12/02	Mon 4/15/02							
14	Install transducer and logg	1 day	Tue 4/16/02	Tue 4/16/02							
15	Tests	7 days	Wed 4/17/02	Thu 4/25/02							
16	Produce Report	7 days	Fri 4/26/02	Mon 5/6/02							
17	Finish	0 days	Mon 5/6/02	Mon 5/6/02							

APPENDIX 6. DRAWINGS